## Gaps and Critical Temperature for Color Superconductivity

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Because of a logarithmic enhancement from soft, collinear magnetic gluons, in dense quark matter the gap for a color superconducting condensate with spin zero depends upon the QCD coupling constant g not as  $\exp(-1/g^2)$ , like in BCS theory, but as  $\exp(-1/g)$ . In weak coupling, the ratio of the transition temperature to the spin-zero gap at zero temperature is the same as in BCS theory. We classify the gaps with spin one, and find that they are of the same order in g as the spin-zero gap.

In cold, dense quark matter, the attractive interaction between quarks of different colors generates color superconductivity [1–10]. In this Letter we discuss in what aspects color superconductivity differs from the classic model of Bardeen, Cooper, and Schrieffer (BCS) [11], and in which aspects it resembles it.

One way in which color superconductivity differs from BCS theory is the dependence of the condensate on the coupling constant. In theories with short-ranged interactions, such as BCS theory, the gap depends upon the coupling constant g as the exponential of  $1/g^2$ . We argued previously, though, that static magnetic interactions are *not* screened to any finite order in g [4,6]. The scattering of quarks near the Fermi surface is then logarithmically enhanced by the emission of collinear, nearly static magnetic gluons, and this changes the gap from an exponential in  $1/g^2$  to one in 1/g.

The explicit value of the gap in weak coupling was first computed by Son [6]. Using an elegant renormalization group argument, he found that there is an instability at a scale  $\phi_0 \sim b_0 \,\mu \,g^{-5} \,\exp(-c_0/g)$ , where  $\mu$  is the quark-chemical potential,  $c_0 = 3\pi^2/\sqrt{2}$ , and  $b_0$  is a pure number. To explicitly compute the magnitude of the spin-zero gap at zero temperature,  $\phi_0$ , it is necessary to solve a gap equation. This was initiated by Son [6]. In this Letter we first extend Son's analysis to estimate the constant  $b_0$ . To the order in g at which we work, all of our results are manifestly gauge invariant. Details will be presented elsewhere [12].

Next, we solve the gap equation at non-zero temperature, T, and show that the critical temperature for the onset of color superconductivity,  $T_c$ , divided by  $\phi_0$  is equal to the value in BCS theory [11],  $T_c/\phi_0 \simeq 0.567 + O(g)$ .

Finally, we classify the gaps for massless fermions with

total spin J=1. There are two types, longitudinal and transverse to the direction of momentum of the quarks in the condensate,  $\phi_1^{\parallel}$  and  $\phi_1^{\perp}$ , respectively. In agreement with Son [6], we find that all spin-one gaps are of the same order as the spin-zero gap:  $\phi_1/\phi_0$  is a pure number of order one.

Our results are of practical importance. Bailin and Love assumed in their original analysis [1] that static magnetic interactions are screened, so that the gaps are BCS-like, and thus tiny,  $\phi_0 \sim 10^{-3}\mu$ . Our results are only valid perturbatively, but if we extrapolate to strong coupling, we find that because the constant  $b_0$  is huge, the gaps can become quite large: as seen in fig. 1,  $\phi_0$  peaks at  $\phi_0 \sim 0.26\,\mu$ , with a big  $T_c \sim 0.15\,\mu$ . At AGS energies, heavy-ion collisions can probe the region of  $\mu \sim 600$  MeV and  $T \sim 100$  MeV. Therefore, by triggering on collisions in which cool, dense nuclear matter is formed, it may be possible to observe color superconductivity.

That J=1 gaps are not exponentially suppressed is important for quark stars. At very high densities, the chemical potential of up, down, and strange quarks are nearly equal, so the J=0 color-flavor locked condensate is surely favored [2]. At intermediate densities, however, because of the large strange quark mass, and the requirement of charge neutrality, these chemical potentials will differ. This suppresses the formation of J=0 gaps, which are predominantly flavor off-diagonal. The J=1 gaps, however, can form between quarks of the same flavor, and will be significant.

We follow the notations and conventions of our previous work [4,5]. For massless quarks there are four types of spin-zero condensates [1,4,5]: right-handed condensates  $\phi_{r,\pm}^{\pm}$ , and left-handed condensates  $\phi_{\ell,\mp}^{\pm}$ . The superscript refers to particles or antiparticles, while the subscript denotes helicity. In perturbation theory, QCD is manifestly chirally symmetric, so that the gap equations for  $\phi_r^{\pm}$  and  $\phi_\ell^{\pm}$  are identical order by order in  $g^2$ . Although the magnitude of the gaps for  $\phi_r$  and  $\phi_\ell$  must then be equal, because they are complex numbers, they differ by an arbitrary phase. This phase represents the spontaneous breaking of parity by a spin-zero, color superconducting gap in an instanton-free regime [4,13].

Without loss of generality, then, we can consider only the right-handed gaps, denoted as  $\phi^+$  and  $\phi^-$ , and take them to be real and positive. Suppressing chiral projectors, and the color and flavor indices [14], the gap function is

$$\Phi(Q) = \phi^{+}(Q) \Lambda^{+}(\mathbf{q}) + \phi^{-}(Q) \Lambda^{-}(\mathbf{q}) . \tag{1}$$

The condensate is formed from a quark, with fourmomentum  $Q = (q^0, \mathbf{q})$ , and a charge conjugate antiquark.  $\Lambda^{\pm}(\mathbf{q}) \equiv (1 \pm \gamma_0 \mathbf{\gamma} \cdot \hat{\mathbf{q}})/2$  are projectors for energy;  $\mathbf{q} = q\,\hat{\mathbf{q}},\,\hat{\mathbf{q}}^2 = 1.$ 

Including the gap, from (15) of [5] the quark propaga-

$$G(Q) = \left[\frac{\Lambda^{+}(\mathbf{q})}{q_0^2 - \epsilon_q^{+2}} + \frac{\Lambda^{-}(\mathbf{q})}{q_0^2 - \epsilon_q^{-2}}\right] (\gamma \cdot Q - \mu \gamma_0), \quad (2)$$

where  $\epsilon_q^{\pm}$  is the energy of the quark relative to the Fermi surface:

$$\epsilon_q^{\pm} \equiv \sqrt{(q \mp \mu)^2 + \phi^{\pm}(Q)^2} \ . \tag{3}$$

The poles with  $\mp \epsilon_q^+$  represent quasiparticles and their holes, those with  $\mp \epsilon_q^-$  quasi-antiparticles and their holes [5]. At the Fermi surface,  $q = \mu$ , it takes very little energy to excite a quasiparticle,  $\epsilon_q^+ = -\phi^+$ , and a lot to excite a quasi-antiparticle,  $\epsilon_q^- \approx -2\mu$ . At one-loop order, from (A35) of [5] the equation for

the gap function  $\Phi(K)$  is

$$\Phi(K) = \frac{2g^2}{3} \frac{T}{V} \sum_{Q} \Delta_{\mu\nu} (K - Q) \gamma^{\mu} G_0^{-}(Q) \Phi(Q) G(Q) \gamma^{\nu}.$$

(4)

Here  $G_0^-(Q) = 1/(\gamma \cdot Q - \mu \gamma_0)$  is the bare propagator for charge-conjugate quarks [14]. To evaluate the Matsubara sum over  $q^0$  we use spectral representations [15].

In the gap equation, the gluon propagator  $\Delta^{\mu\nu}$  includes the effects of "hard dense loops" (HDL) [15]. The basic parameter of the HDL Lagrangian is the gluon "mass",  $m_q$ ; for  $N_c$  colors and  $N_f$  flavors of massless quarks,

$$m_g^2 = N_f \frac{g^2 \mu^2}{6\pi^2} + \left(N_c + \frac{N_f}{2}\right) \frac{g^2 T^2}{9} .$$
 (5)

For the time being we take strict Coulomb gauge for the HDL propagator. HDL corrections can be neglected for the quark propagator and the quark-gluon vertex, as the quark lines are hard,  $q \sim \mu$ .

We solve the gap equation by including the effects of the superconducting state in the simplest possible way for the quark, Eq. (2), and not at all for the gluon. This is reasonable in weak coupling, because the scale of the condensate,  $\phi_0 \sim \mu \exp(-c_0/g)$ , is much smaller than either  $\mu$  or  $m_g \sim g\mu$  [16].

As in strong coupling BCS theory [11],  $\Phi(K)$  has an imaginary part, but for small g this can be neglected in QCD [17]. Consequently, the only values of the gap functions  $\phi^{\pm}(Q)$  which enter into the gap equation are those on either the quasiparticle mass shell,  $\phi^+(\pm \epsilon_q^+, q)$ , or the quasi-antiparticle mass shell,  $\phi^-(\pm \epsilon_q^-, q)$ .

Gap equations for  $\phi^{\pm}(\epsilon_k^{\pm}, k)$  are derived from (4) via projection with  $\Lambda^{\pm}(\mathbf{k})$ . As is typical in models of superconductivity [11], the dominant terms arise from the quasiparticle poles. These correspond physically to scattering of quarks near the Fermi surface. As this involves little energy transfer between the quarks, it suffices to use the nearly static limit of the gluon propagator.

With these approximations, denoting  $\epsilon_k^+ = \epsilon_k$ , the gap equation for  $\phi(k) \equiv \phi^+(\epsilon_k, k)$  becomes [12]

$$\phi(k) = \frac{g^2}{36\pi^2} \int_{\mu-\delta}^{\mu+\delta} \frac{\mathrm{d}q}{\epsilon_q} \frac{1}{2} \ln\left(\frac{b^2 \mu^2}{\epsilon_q^2 - \epsilon_k^2}\right) \tanh\left(\frac{\epsilon_q}{2T}\right) \phi(q) ,$$
(6)

$$b = \frac{b_0}{g^5} = b_t^2 b_l^3 b_0' = 256 \pi^4 \left(\frac{2}{g^2 N_f}\right)^{5/2} b_0', \qquad (7)$$

where  $b_{\rm t}=4\sqrt{2}\,\mu/(\sqrt{3\pi}\,m_g),$  and  $b_{\rm l}=2\,\mu/(\sqrt{3}\,m_g).$  The logarithm  $\sim \ln[1/(\epsilon_q^2 - \epsilon_k^2)]$  arises from the cut term in the spectral density of a nearly static transverse gluon [4,6]. In the gap equation, there are also terms  $\sim \ln(1/q)$ which arise from the non-static transverse gluons and from static longitudinal gluons; these produce the constants  $b_t$  and  $b_l$ , respectively. In addition, there are terms  $\sim 1$  in the gap equation which contribute to the constant  $b_0'$ ; we did not compute these terms. In deriving (6) we assume that  $\epsilon_k, \epsilon_q < \mu$ , so we introduce a cut-off  $\delta$  on the q-integration; the final result is independent of  $\delta$ .

At T=0, an approximate solution of (6) is [12]

$$\phi(k) = \phi_0 \sin(\bar{g} y_k) , \qquad (8)$$

$$\bar{g} \equiv \frac{g}{3\sqrt{2}\pi} , \quad y_k \equiv \ln\left(\frac{2b\mu}{|k-\mu| + \epsilon_k}\right) ,$$

where  $\phi_0$  denotes the value of the condensate at the Fermi surface,  $k = \mu$ . (This is similar, but not identical to the solution of [6,8,10].) As  $y_{\mu} = \ln(2b\mu/\phi_0)$ , Eq. (8) requires  $\bar{g} y_{\mu} = \pi/2, i.e.,$ 

$$\phi_0 = 2 b\mu \exp\left(-\frac{\pi}{2\bar{g}}\right) . \tag{9}$$

Our results for  $c_0$  and the prefactor  $1/g^5$  are in agreement with Son [6]. The constants  $b_t$  and  $b_l$  are the same found in an independent analysis by Schäfer and Wilczek [8]; see also Hong et al. [10].

In BCS-like theories with zero-ranged interactions, such as Nambu–Jona-Lasinio (NJL) models [2], all particle pairs around the Fermi surface contribute with equal weight to build up the BCS-logarithm, so that the gap function is constant:  $\sim g^2 \int \mathrm{d}q/\epsilon_q \simeq g^2 \ln(2\delta/\phi_0)$ , with solution  $\phi_0 \sim 2\delta \exp(-1/g^2)$ . In a model where fermions interact with scalar bosons of mass  $M_s \sim g\mu$  [5], scattering of particle pairs through small angles is favored. The collinear singularity is cut off by  $M_s \neq 0$ , so that

logarithmic factors of  $\sim \ln(\mu/M_s) \sim \ln(1/g)$  appear in the gap equation.

In QCD, the scattering of quark pairs through small angles is again favored. If the exchanged gluon is electric, the collinear singularity is cut off by the Debye mass,  $\sqrt{3} m_q$ . This produces  $\ln(1/g)$  terms which contribute to the prefactor  $1/g^5$  in b, Eq. (7). If the exchanged gluon is magnetic, the collinear singularity is only cut off by the difference in energies between the incoming and outgoing pairs. In the gap equation (6), this generates the logarithmic enhancement factor  $\sim \ln[1/(\epsilon_q^2 - \epsilon_k^2)]$ . The dependence of the gap function on  $\epsilon_k$  is then not negligible. Quasiparticles with momenta exponentially close to the Fermi surface,  $\epsilon_q \sim b\mu \exp(-c/g)$ , dominate the integral, with a contribution which is enhanced by  $\ln(b\mu/\epsilon_q) \sim c/g$ . The gap function  $\phi(q)$  is weighted towards these pairs, as  $\phi(q)/\phi_0 \sim \sin(\pi c/2c_0) \sim 1$ . For quasiparticles which are not exponentially close to the Fermi surface,  $\epsilon_q \sim \mu$  and  $c \sim g$ , the gap function is down by  $\phi(q)/\phi_0 \sim g$  [17].

The temperature dependence of the condensate can be computed from Eq. (6) as follows. We assume that the temperature T is of the order of the gap at zero temperature,  $\phi_0$ . Let us introduce a dimensionless parameter  $\kappa \gg 1$ , and divide the integration region into  $\epsilon_q \geq \kappa \phi_0$  and  $\epsilon_q < \kappa \phi_0$ . Away from the Fermi surface,  $\epsilon_q \gg \phi_0$ , the Fermi–Dirac distribution becomes a Boltzmann distribution, so  $\tanh(\epsilon_q/2T) \simeq 1$ , and thermal effects are negligible. Near the Fermi surface, the thermal factor  $\tanh(\epsilon_q/2T)$  cuts off any singularity, even at the critical temperature,  $T_c$ , when  $\phi(q) \to 0$ . Then the gap function is the same as (8) for  $\epsilon_k \gg \kappa \phi_0$ , and a constant for  $\epsilon_k \ll \kappa \phi_0$ . Matching the two regions at  $\kappa \phi_0$ , and then sending  $\kappa \to \infty$ , we derive the condition

$$\int_0^\infty d|q - \mu| \left[ \frac{1}{\epsilon_q} \tanh\left(\frac{\epsilon_q}{2T}\right) - \frac{1}{\epsilon_q^0} \right] = 0 , \qquad (10)$$

where  $\epsilon_q = \sqrt{(q-\mu)^2 + \phi^2(T)}$ , with  $\phi(T)$  the gap at the Fermi surface at a temperature T, and  $\epsilon_q^0 = \sqrt{(q-\mu)^2 + \phi_0^2}$ . This is correct to leading order in g. Equation (10) implicitly determines the function  $\phi(T)/\phi_0$ ; it is identical to that obtained in BCS theory in weak coupling [11,12]. In particular, the ratio of the critical temperature to the zero-temperature gap is the same as in BCS,  $T_c/\phi_0 = \zeta/2 + O(g)$ , where the constant  $\zeta = 2 e^{\gamma}/\pi \simeq 1.13$ . ( $\gamma \simeq 0.577$  is the Euler-Mascheroni constant.)

Following the classification of [4,5], for massless quarks a spin-one condensate has the form

$$\sum_{h,e} \left( \phi_h^{\parallel e}(Q) \cdot \hat{\mathbf{q}} + \phi_h^{\perp e}(Q) \cdot \mathbf{P}(\mathbf{q}) \cdot \boldsymbol{\gamma} \right) \mathcal{P}_h \, \Lambda^e(\mathbf{q}) \;, \quad (11)$$

where the sum runs over chiralities,  $h = r, \ell$ , and energies,  $e = \pm$ .  $\mathcal{P}_{r,\ell} = (1 \pm \gamma_5)/2$  is the chiral projector, and

 $\mathbf{P}(\mathbf{q}) = \mathbf{1} - \hat{\mathbf{q}}\hat{\mathbf{q}}$  a projector onto the subspace orthogonal to q. Because a spin-one condensate is a three-vector, there are 12 types of condensates, four longitudinal,  $\phi_{1,h}^{\parallel e} \equiv \phi_h^{\parallel e} \cdot \hat{\mathbf{q}}$ , and eight transverse,  $\phi_{1,h}^{\perp e} \equiv \phi_h^{\perp e} \cdot \mathbf{P}(\mathbf{q})$ . This classification is equivalent to that of [1]. While the spin-zero gaps are symmetric in the simultaneous interchange of color and flavor indices [1,4,5], the longitudinal gaps  $\phi_{1,h}^{\parallel e}$  are antisymmetric. The transverse gaps fulfill a more complicated relationship,  $(\phi_{r,\ell}^{\perp\pm})^T = -\phi_{\ell,r}^{\perp\mp}$ . The spin-zero gaps and the longitudinal spin-one gaps do not mix quarks of different chirality; the transverse spin-one gaps do, and thus break chiral symmetry. The gap equations can be constructed as in the spin-zero case [12]. We find that both the longitudinal as well as the transverse gaps fulfill the same gap equation as the spin-zero gaps, with identically the same solution as in (9), except that the constant analogous to  $b'_0$  (which we do not compute) may differ.

In the static limit, gauge dependent terms in the gluon propagator  $\Delta^{\mu\nu}(P)$  are  $\sim p^{\mu}p^{\nu}/p^2$  [15]. These terms contribute to the gap equation, but neither to  $c_0$ , the power of g in the prefactor,  $b_t$ , nor  $b_l$ . They do appear to contribute to the undetermined constant  $b'_0$ , but we suggest that in the end,  $b'_0$  is gauge invariant. There are other effects which contribute to  $b'_0$  [12]. One-loop diagrams with a soft, transverse HDL gluon propagator renormalize the quark [18] and gluon wave functions, and the quark-gluon vertex. Other contributions arise from the influence of the condensate on the gluon propagator [16], and the admixture of quasi-antiparticle modes in the quasiparticle gap equation. It is important to calculate  $b'_0$ , since its numerical value determines exactly which patterns of symmetry breaking are favored.

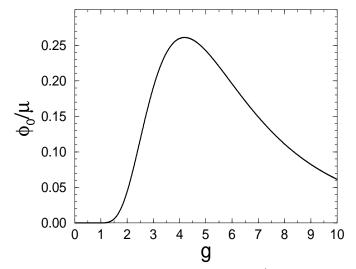


FIG. 1.  $\phi_0/\mu$  as function of g, for  $b'_0 = 1$ .

While the results which we have derived are rigorously valid only in weak coupling, it is interesting to plot  $\phi_0/\mu$  as a function of g, fig. 1. We take  $N_f=2$  (note that

 $b_0 \sim 1/N_f^{5/2}$ ). From (7),  $\phi_0/\mu$  is proportional to  $b_0'$ ; in fig. 1 we set this undetermined constant equal to 1. Equation (9) has the form of a semiclassical tunneling probability, including a prefactor from five zero modes. Because of the "zero modes", the gap function peaks at a value of  $\phi_0/\mu \sim 0.26$  when  $g \sim 4.2$ .

Extending the picture of Schäfer and Wilczek [3], we view quark matter as a color superconducting "liquid", and hadronic matter as a color superconducting "vapor". From [4] there is a first-order phase transition between these liquid and vapor phases at  $\mu = \mu_c$  and T = 0. Then perhaps at several times nuclear matter density, the liquid phase occurs at the maximum of  $\phi_0/\mu$ , and the vapor phase at larger g, providing a qualitative explanation for the smallness of the analogous gaps in hadronic matter [2].

We conclude by using (4) to estimate the validity of perturbation theory. Perturbative calculations break down when  $m_g \simeq \mu$  or T. For  $N_c = 3$  and  $N_f = 2$ , at  $T \neq 0$  and  $\mu = 0$ ,  $m_g = T$  when the QCD fine structure constant is tiny,  $\alpha_s \equiv g^2/4\pi \sim 0.18$ . In contrast, at  $\mu \neq 0$  and T = 0,  $m_g = \mu$  when  $\alpha_s$  is much larger,  $\alpha_s \sim 2.4$ . This suggests to us that while perturbation theory is not a good approximation for hot quark-gluon matter [19], it may well be a reasonable guide to understanding dense quark matter, as long as it is cold,  $T < 0.3\mu$ .

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- [1] D. Bailin and A. Love, Phys. Rep. 107, 325 (1984).
- [2] M. Alford, K. Rajagopal, and F. Wilczek, Phys. Lett. B422, 247 (1998); M. Alford, K. Rajagopal, and F. Wilczek, Nucl. Phys. B537, 443 (1999); R. Rapp, T. Schäfer, E.V. Shuryak, and M. Velkovsky, Phys. Rev. Lett. 81, 53 (1998); hep-ph/9904353; N. Evans, S.D.H. Hsu, and M. Schwetz, Nucl. Phys. B551, 275 (1999); Phys. Lett. B449 281, (1999); J. Berges and K. Rajagopal, Nucl. Phys. B538, 215 (1999); T. Schäfer and F. Wilczek, Phys. Lett. B450, 325 (1999); G.W. Carter and D. Diakonov, Phys. Rev. D 60, 016004 (1999); K. Langfeld and M. Rho, hep-ph/9811227; M. Alford, J. Berges, and K. Rajagopal, hep-ph/9903502.
- [3] T. Schäfer and F. Wilczek, Phys. Rev. Lett. 82, 3956 (1999); hep-ph/9903503.
- [4] R.D. Pisarski and D.H. Rischke, nucl-th/9811104, to ap-

- pear in Phys. Rev. Lett.
- [5] R.D. Pisarski and D.H. Rischke, nucl-th/9903023, to appear in Phys. Rev. D.
- [6] D.T. Son, Phys. Rev. D 59, 094019 (1999).
- [7] E. Shuster and D.T. Son, hep-ph/9905448.
- [8] T. Schäfer and F. Wilczek, hep-ph/9906512.
- [9] D.K. Hong, hep-ph/9812510, hep-ph/9905523.
- [10] D.K. Hong, V.A. Miransky, I.A. Shovkovy, and L.C.R. Wijewardhana, hep-ph/9906478.
- [11] J.R. Schrieffer, Theory of Superconductivity (New York, W.A. Benjamin, 1964); D.J. Scalapino, in: Superconductivity, ed. R.D. Parks, (New York, M. Dekker, 1969), p. 449ff
- [12] R.D. Pisarski and D.H. Rischke, manuscript in preparation.
- [13] R.D. Pisarski and D.H. Rischke, nucl-th/9906050.
- [14] The interaction between two quarks contains two pieces, which are symmetric or antisymmetric in the color indices of the fundamental representation. The antisymmetric representation is attractive to lowest order in g; for an  $SU(N_c)$  gauge theory, the coefficient in (4) is  $g^2(N_c+1)/(2N_c)$  [7]. When  $N_c=3$ , the antisymmetric representation is the color  $\overline{\bf 3}$  representation, the symmetric the color  $\bf 6$ . Fermi statistics for a J=0 gap imposes constraints which require the number of massless flavors,  $N_f \geq 2$  [4]. The form of the quark propagator in (2) is only valid when  $N_f=2$ , and the  $(SU(3)_c, SU_{r,\ell}(N_f))$  representation is  $(\overline{\bf 3}, {\bf 1})$  [4]. When  $N_f=3$ , the  $(\overline{\bf 3}, \overline{\bf 3})$  representation mixes with the  $({\bf 6}, {\bf 6})$  [4]. This mixing only affects the gap equation to higher order  $\sim \phi_0^2$ , which is negligible in weak coupling.
- [15] M. Le Bellac, Thermal Field Theory (Cambridge, Cambridge University Press, 1996).
- [16] Due to infrared singular factors, the effective action for the condensate is  $\sim |D_{\mu}\phi|^2/\phi_0^2$ , so that the (true) gluon mass from color superconductivity is not  $m_{super} \sim g\phi_0$ , as one would naively expect, but much larger,  $m_{super} \sim g\mu$ . (We thank T. Schäfer for discussions on this point.) To the order at which we work, this is irrelevant for the gap equation, because the dominant momenta are  $\gg \phi_0$ , and on that scale, corrections from the condensate are small,  $\sim g^2\phi_0/q$  at large  $q\gg\phi_0$ .
- [17] From (6) the imaginary part of  $\phi(k)$  arises from the cut in the logarithm for  $\epsilon_q < \epsilon_k$ ,  $\operatorname{Im} \phi(k) \sim g^2 \int_{\phi_0}^{\epsilon_k} \mathrm{d}\epsilon_q/\epsilon_q \ \phi(q) \simeq g^2 \ln(\epsilon_k/\phi_0)\phi_0$ . Taking  $\epsilon_k \sim b\mu \exp(-c/g)$ , momenta exponentially close to the Fermi surface occur when  $c \sim 1$ . In this region, the imaginary part of the gap function,  $\operatorname{Im} \phi(k) \sim g(c_0 c)\phi_0$ , is down by g relative to the real part,  $\operatorname{Re} \phi(k) \sim \sin(\pi c/2c_0)\phi_0$ . Away from the Fermi surface,  $\epsilon_k \sim \mu$ , so  $c \sim g$ , and  $\phi(k)$  is strongly damped, with the real and imaginary parts of comparable magnitude,  $\operatorname{Re} \phi(k) \sim \operatorname{Im} \phi(k) \sim g\phi_0$ .
- [18] T. Holstein, R.E. Norton, and P. Pincus, Phys. Rev. B 6, 2649 (1973).
- [19] J.O. Andersen, E. Braaten, and M. Strickland, hep-ph/9902327, hep-ph/9905337; J.-P. Blaizot, E. Iancu, and A. Rebhan, hep-ph/9906340.